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# Transient temperature rises in a sandwich-structured composite induced by a dithering or rotating laser beam

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## ABSTRACT

Numerical solutions are provided for a transient temperature rise in a composite sandwich structure that is induced by a dithering or rotating laser beam. Here we consider using E-glass or C-glass fibers as skin materials whereas the core can be selected from a variety of materials including balsa wood, honeycomb, cork and C-foam. Our results provide insights in engineering sandwich-structured composite as heat shields. More specifically, it is shown that (1) carbon fibers are better skin materials than E-glass fibers; (2) sandwich-structured composite with carbon fiber skin and honeycomb core is most promising.

**Keywords:** rotating or dithering Gaussian laser beam; non-homogeneous heat equation; composite sandwich structure

## 1. INTRODUCTION

Direct energy laser weapons deliver high-intensity beams to an object and can quickly destroy or burn it. In order to protect important objects, such as Navy ships and missiles, one possible counter measure is to use materials of composite sandwich structure to enhance damage tolerance of the object or let the object rotate or shake to reduce the maximum temperature rise. The main purpose here is to investigate the temperature rise of a composite sandwich structure induced by a rotating or dithering laser beam and find ways to reduce or avoid laser weapon damages.

The study of the temperature rise induced by a laser beam is not a new subject. A lot of research have been devoted to the theoretical and numerical modeling of temperature profiles induced by laser radiation in solids (Araya and Gutierrez, 2006; Bertolotti and Sibilia., 1981; Burgener and Reedy, 1982; Calder and Sue, 1982; Cline and Anthony, 1977; Lax, 1977 and 1978; Moody and Hendel, 1982; Sanders, 1984; Sistaninia et al., 2009) and temperature distributions in a two-layer structure by a scanning laser beam (Burgener and Reedy, 1982). Recently we have given a detailed study of the temperature rise induced by a rotating or dithering laser beam on a semi-infinite domain (Zhou, 2011a), a solid with finite geometry (Zhou and Tan, 2011; Tan and Zhou, 2012), and a two-layer structure (Zhou, 2011b; Zhou, 2012).

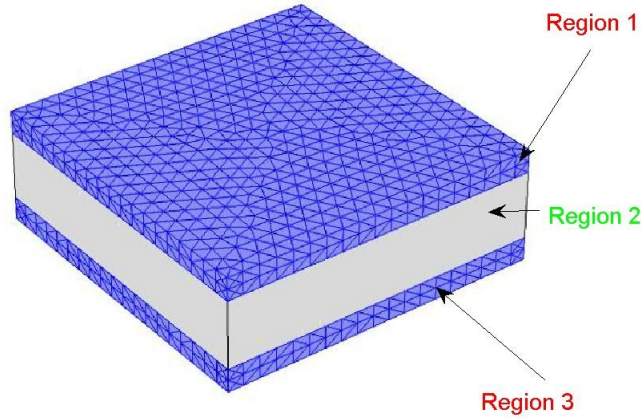
In this paper we extend our earlier studies to a sandwich-structured composite, which is a special class of composite materials and is fabricated by attaching two stiff but thin skins to a lightweight but thick core. The core material normally has low strength, but its higher thickness provides the sandwich-structured composite with high bending stiffness and overall low density. Thicker core material leads to stronger sandwich-structured composite. Commonly-used core materials include balsa wood, honeycombs, cork and C-foam whereas widely-used skin materials are E-glass fibers and Carbon fibers. Sandwich-structured composite materials are becoming popular in high-performance products that need to be lightweight, yet strong enough to endure harsh loading conditions such as boat hulls, bicycle frames, sports car bodies and spacecraft components. We would like to investigate the temperature rise induced by a dithering or rotating laser beam on a sandwich-structured composite and compare the performance of different materials when used as heat shields.

We organize our paper as follows. In Sections 2-3 we present the modeling and numerical solutions of the temperature distributions in a three-dimensional composite sandwich structure induced by a dithering or rotating laser beam, respectively. Finally, conclusions are drawn in Section 4.

## 2. MATHEMATICAL FORMULATIONS

We first construct a time-dependent model of a three-dimensional composite sandwich structure made of one core material sandwiched between two thin layers of other materials. Laser beams are shone on the top surface of the sandwich structure. Figure 1 depicts a composite sandwich structure. We divide the finite structure into three regions: Region 1 is a thin layer of thickness  $d_1$  in the  $z$  direction, which is on a substrate of

dissimilar material of thickness  $d_2$  (Region 2), and Region 3 is another thin layer of thickness  $d_3$  at the bottom of the structure. The materials in both Region 1 and Region 3 are the same.



**Figure 1: A schematic diagram of a three-dimensional composite sandwich structure.**

Mathematically, the temperature distribution in the composite sandwich structure can be modeled as follows. In Region 1, the governing equation is

$$\frac{\partial u_1}{\partial t} = \alpha_{T,1} \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) + \frac{\alpha_{T,1}}{K_{T,1}} q(x, y, z, t), \quad (1)$$

where  $u_1(x, y, z, t)$  denotes the temperature rise at position  $(x, y, z)$  and time  $t$ ,  $\alpha_{T,1}$  is the thermal diffusivity of the material in Region 1,  $K_{T,1}$  is the thermal conductivity, and  $q(x, y, z, t)$  is the energy distribution of the moving laser beam. In the simulations presented in this paper we model the distribution of a dithering or rotating Gaussian beam as

$$q(x, y, z, t) = f(x, y, t) \delta(z - z_0). \quad (2)$$

Here  $f(x, y, t)$  is given by

$$f(x, y, t) = \frac{I_0}{2\pi d^2} \exp \left[ -\frac{(x - x_c(t))^2 + (y - y_c(t))^2}{2d^2} \right], \quad (3)$$

$$x_c(t) = x_0 + a \cos \frac{2\pi t}{T}, \quad x_0 = \frac{L_x}{2},$$

$$y_c(t) = y_0 + b \sin \frac{2\pi t}{T}, \quad y_0 = \frac{L_y}{2},$$

where  $(x_c(t), y_c(t), z_0)$  is the position of the dithering or rotating Gaussian beam,  $(x_0, y_0, z_0)$  is the initial position of the laser beam,  $I_0$  is the intensity of the laser beam,  $r_0$  is the effective radius of the laser beam,  $k$  is a constant used for the Gaussian model and  $\delta(z - z_0)$  is the Dirac delta function. Equation (3) represents a

rotating laser beam when  $a = b$  and a dithering beam when either  $a$  or  $b$  is zero. When  $a = b = 0$ , equation (3) simply represents a non-moving Gaussian beam. In Region 2, the equation for the temperature rise  $u_2(x, y, z, t)$  where the material has the thermal diffusivity  $\alpha_{T,2}$  and thermal conductivity  $K_{T,2}$  is

$$\frac{\partial u_2}{\partial t} = \alpha_{T,2} \left( \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2} \right). \quad (4)$$

In Region 3, the equation for the temperature rise  $u_3(x, y, z, t)$  is

$$\frac{\partial u_3}{\partial t} = \alpha_{T,3} \left( \frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2} \right). \quad (5)$$

At the interfaces between the materials, we require that the energy is conserved for heat flow across the interfaces:

$$\begin{aligned} K_{T,1} \frac{\partial u_1}{\partial z} &= K_{T,2} \frac{\partial u_2}{\partial z} \quad \text{at the interface between Regions 1 and 2} \\ K_{T,2} \frac{\partial u_2}{\partial z} &= K_{T,3} \frac{\partial u_3}{\partial z} \quad \text{at the interface between Regions 2 and 3} \end{aligned} \quad (6)$$

The initial condition for  $u_1(x, y, z, t)$ ,  $u_2(x, y, z, t)$ ,  $u_3(x, y, z, t)$  is zero which assumes that the composite sandwich structure has the same temperature as the ambient initially. The boundary conditions at the air/material interface impose that the sandwich structure is insulated at the edges. The boundary conditions reflect the assumption that no energy escapes into the ambient at the air/material interface. This is a good approximation for most materials under consideration because heat flow by conduction through the material is usually much bigger than heat loss by radiation or convection at the air/material interface.

For the top (Region 1) and bottom (Region 3) skins of the composite sandwich structure, the materials are either *E-glass fibers* or *carbon fibers*. E-glass or electrical grade glass is a lightweight, extremely strong, low-cost and non-flammable material. It is also relatively insensitive to moisture and has good electrical insulation. Common uses of E-glass include surfboards, sports cars, boats, flat roofs, water tanks and pipes. Compared to E-glass fibers, carbon fibers are relatively expensive similar fibers. They have high flexibility, high tensile strength, low weight, high temperature tolerance and low thermal expansion. Carbon fibers are very popular in aerospace, civil engineering, military, and automobile. An E-glass/carbon fiber component is typically of a thin "shell" construction, sometimes filled on the inside with structural core, as in the case of surfboards.

As for the core (Region 2), we choose several materials: *balsa wood*, *Nomex honeycomb*, *cork* and *C-foam*. Here we give a very brief introduction to each of these four materials. (1) Balsa wood is often used as a core material in composites, for example, many wind turbines are made partially of balsa wood. It is also used in the floor pan of the luxury sports car Chevrolet Corvette Z06 sandwiched between two sheets of carbon fiber-reinforced polymer. (2) Nomex honeycomb is a lightweight, flame resistant material originally developed in the early 1960s by Dupont Chemical for the aviation industry. It has excellent thermal, chemical, and radiation resistance and is widely used in firefighting equipment. (3) Cork is an impermeable, buoyant material that is extracted primarily from the Cork Oak trees. Cork is used as bottle stoppers, floor and wall tiles, bricks for the outer walls of houses, and a core material in sandwich composite construction. It is also used in spacecraft heat shields. (4) C-foam is a structural material that is inexpensive, lightweight, fire-resistant, and impact-absorbing. C-foams are currently being developed for many uses including fire-resistant ship decking, noise and impact mitigation for aircraft, structural panels and firewalls for automobiles.

The thermal properties of the skin and core materials are listed in Table 1 (Wang et al. 2007; Silva et al. 2005).

### 3. NUMERICAL SOLUTIONS

In this section we will compare the performance of each core material combined with two different skin materials: E-glass fibers or Carbon fibers. We select the commercial software COMSOL to solve the equations (1)-(6) with the given initial and boundary conditions. COMSOL is a finite-element-based software which can be exploited to solve our model equations efficiently.

We consider a sandwich structure where the core has thickness  $6mm$  and cross section dimension  $304.8mm \times 304.8mm$  and the top and bottom skins are  $0.5mm$  thick. First we shine a Gaussian beam on the top surface of the sandwich structure where  $I_0 = 500W, d = 0.0025m, a = b = 0, L_x = L_y = 0.3048m$  in equation (3).

**Table 1: Thermal properties of various materials**

Property Name	E-glass fiber	Carbon fiber	Balsa wood	Nomex honeycomb	Cork	C-foam
Heat Capacity (or Specific Heat) $C_p$ (unit: $J/(kg \cdot K)$ )	900	676	1700	1172 ~1340	350	1260
Density $\rho$ (unit: $Kg/(m^3)$ )	2580	1760	130	1100	150	480
Thermal Conductivity $K_T$ (unit: $W/(m \cdot K)$ )	1.3	84	0.03 ~ 0.07	0.175	0.045	0.033 ~0.050
Thermal Diffusivity $\alpha_T = \frac{K_T}{\rho \cdot C_p}$ (unit: $m^2/s$ )	$5.5986 \times 10^{-7}$	$7.0602 \times 10^{-5}$	$1.3575 \times 10^{-7} \sim 3.1674 \times 10^{-7}$	$1.1872 \times 10^{-7} \sim 1.3574 \times 10^{-7}$	$8.5714 \times 10^{-7}$	$5.4563 \times 10^{-8} \sim 8.2672 \times 10^{-8}$

Figures 2-5 show the temperature rises in the sandwich structure with different materials at time  $t = 6s$ . More specifically, in the left column of Figure 2 the temperature rise on an E-glass/balsa wood composite is shown whereas the right column depicts the temperature rise on an E-glass/honeycomb composite. Hitting by a Gaussian beam for 6 seconds, the E-glass/balsa wood composite reaches a maximum temperature of about  $3.7 \times 10^4 K$  on the incident face whereas the E-glass/honeycomb composite has a maximum temperature of about  $2.5 \times 10^4 K$  on the incident face. This indicates that using different core materials can make a big difference in reducing maximum temperature rise. Figure 3 shows the temperature rise of the E-glass/cork composite and the E-glass/C-foam composite after hitting by a Gaussian beam for 6 seconds. The E-glass/cork composite obtains a maximum temperature  $4.0 \times 10^4 K$  on the incident face whereas the maximum incident face temperature of E-glass/C-foam rises to  $3.4 \times 10^4 K$ .

Figures 4-5 are similar to Figures 2-3, where the skin material is replaced by carbon fibers. From Figures 4-5 we can see that the maximum temperature rise on the incident face is reduced significantly now in each composite sandwich structure (all about 4000K).

Of practical interest is the maximum temperature rise on the top/bottom surfaces (incident face and rear face) and the middle cross-section of the sandwich structure. We document our numerical results in Table 2. From Table 2, one can see that Carbon fibers are better skin materials than E-glass fibers. Moreover, the Carbon fiber/Nomex honeycomb sandwich structure has the best performance with smallest value of the maximum temperature rise.



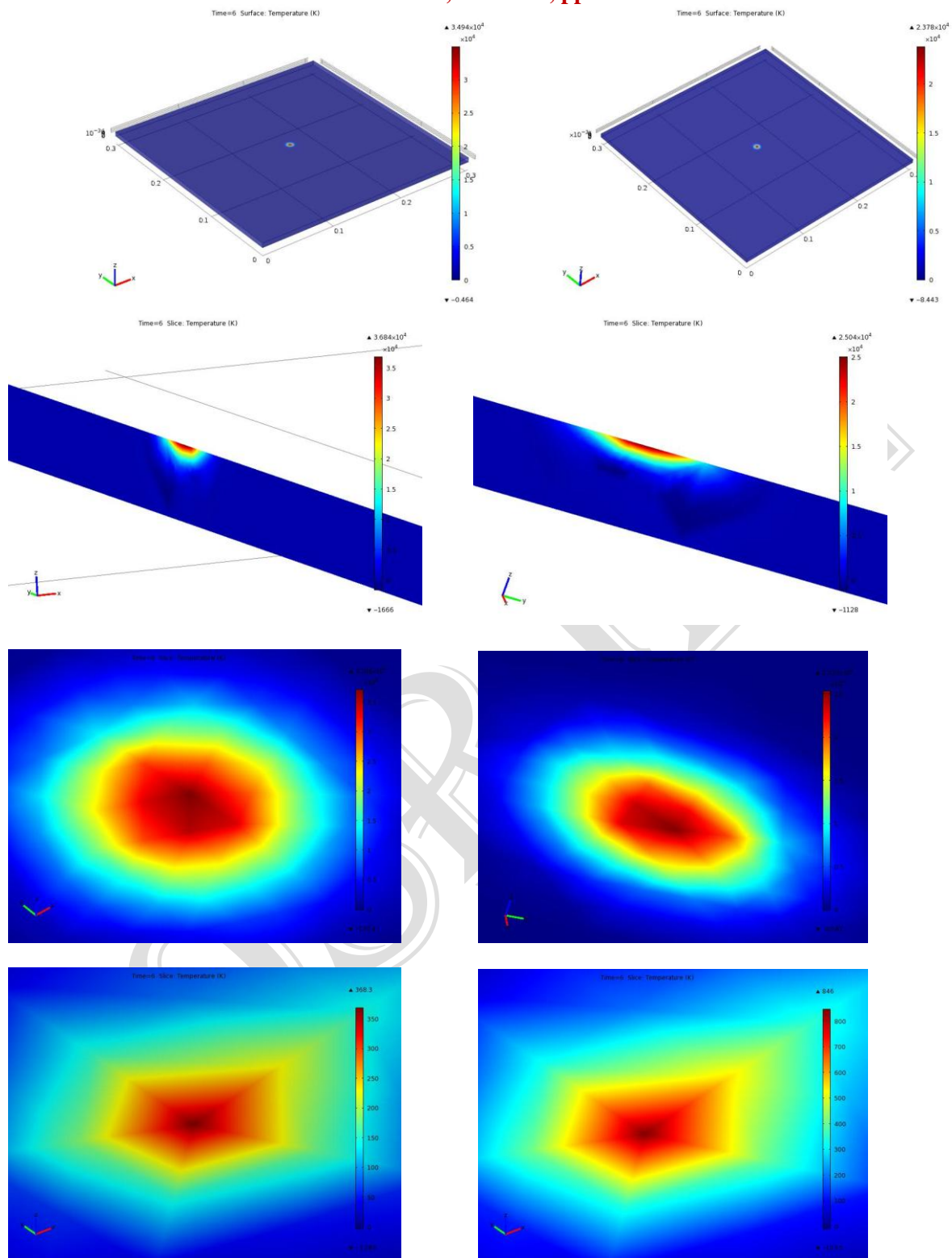


Figure 2: The temperature rise of a sandwich structure with E-glass fiber skins induced by a Gaussian beam. Left column: Balsa wood core; Right column: Nomex honeycomb core. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.

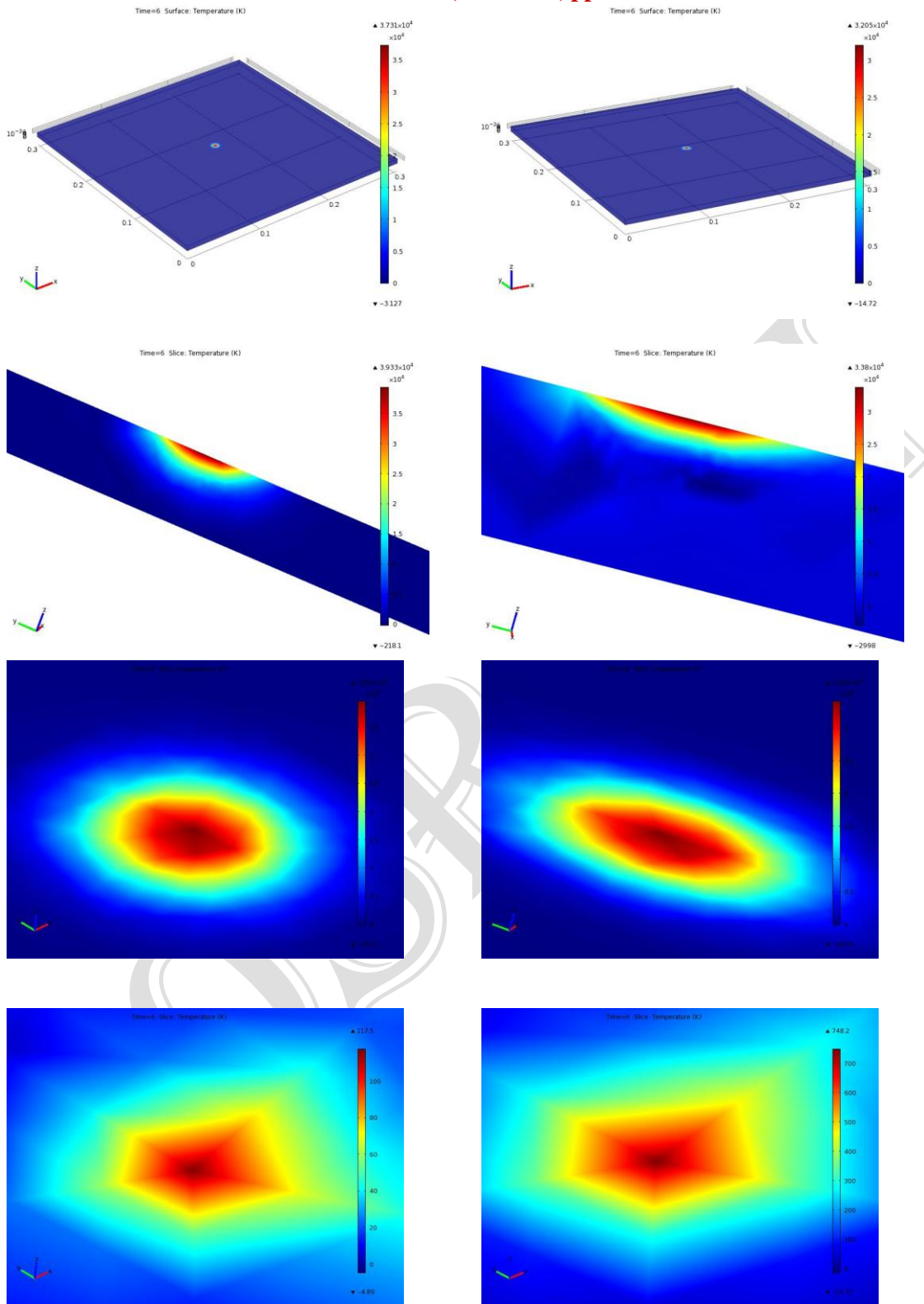


Figure 3: The temperature rise of a sandwich structure with E-glass fiber skins induced by a Gaussian beam. Left column: cork core; Right column: C-foam core. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.

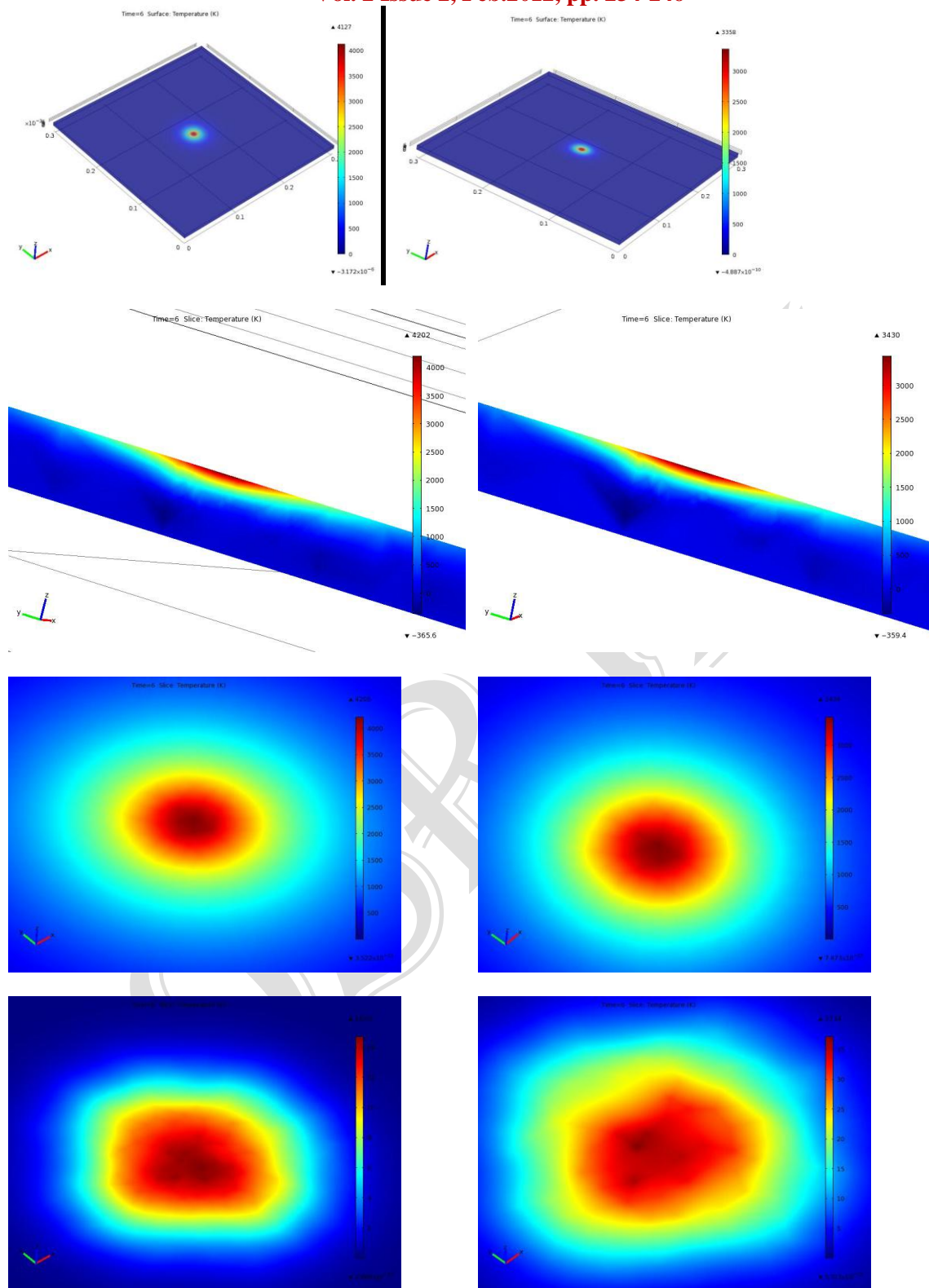


Figure 4: The temperature rise of a sandwich structure with Carbon fiber skins induced by a Gaussian beam. Left column: Balsa wood core; Right column: Nomex honeycomb core. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.



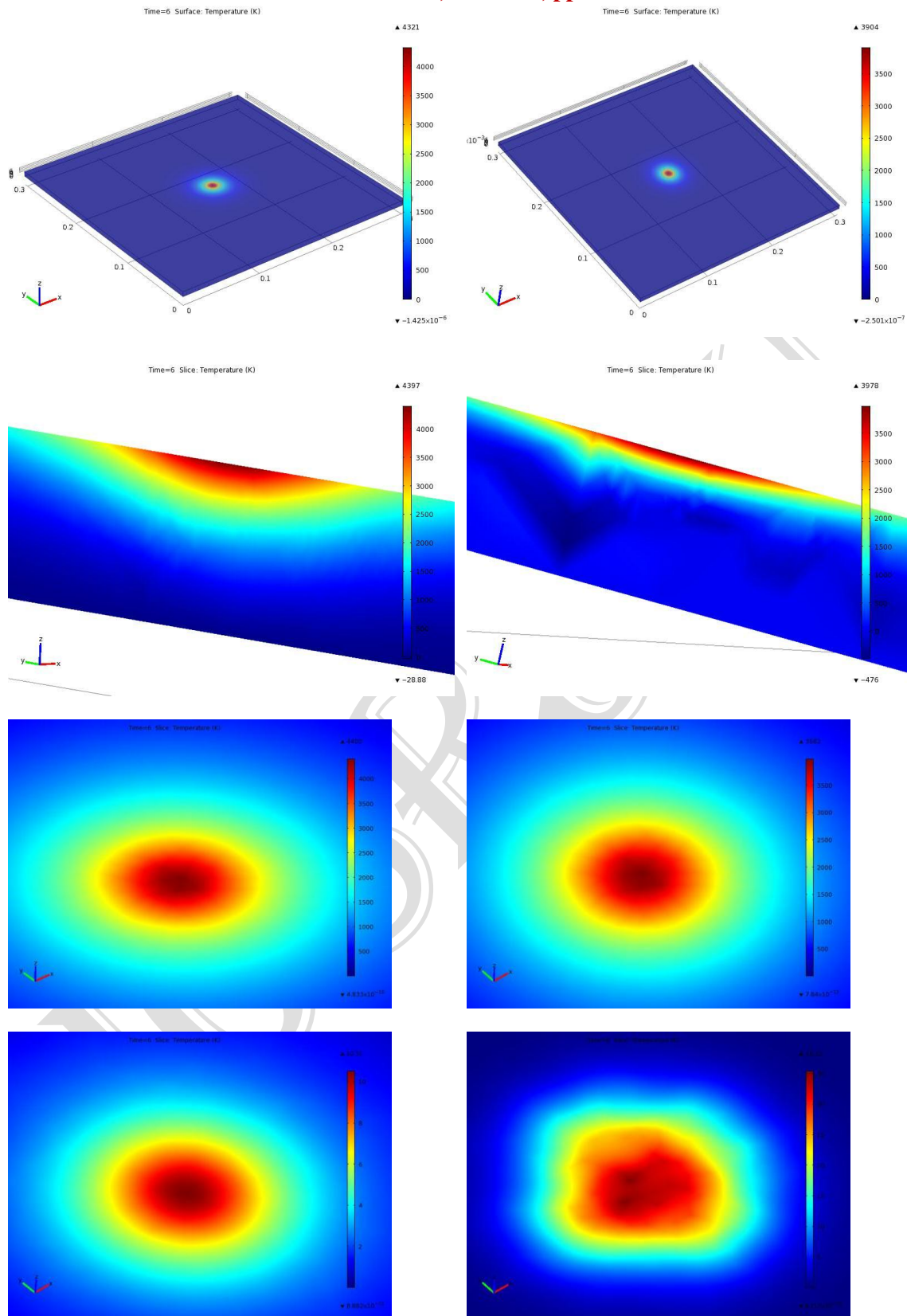


Figure 5: The temperature rise of a sandwich structure with Carbon fiber skins induced by a Gaussian beam. Left column: cork core; Right column: C-foam core. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.

Table 2: Maximum temperature rise on a sandwich structure induced by a Gaussian laser beam

Skin material	Core material	$T_{\max}$ on the incident face (K)	$T_{\max}$ on the middle cross-section (K)	$T_{\max}$ on the rear face (K)
E-glass fiber	Balsa wood	$3.708 \times 10^4$	3993	368
E-glass fiber	Nomex honeycomb	$2.536 \times 10^4$	2293	846
E-glass fiber	Cork	$3.954 \times 10^4$	7463	117
E-glass fiber	C-foam	$3.409 \times 10^4$	3228	748
Carbon fiber	Balsa wood	4206	612	14
Carbon fiber	Nomex honeycomb	3434	411	37
Carbon fiber	Cork	4400	1294	10
Carbon fiber	C-foam	3982	472	35

Next we study the temperature rise induced by a dithering laser beam. In formula (3) we choose  $I_0 = 500W$ ,  $d = 0.0025m$ ,  $a = 0$ ,  $b = 0.05m$ ,  $L_x = L_y = 0.3048m$ ,  $T = 1s$ . In Figure 6 we show the temperature rise on the sandwich structure induced by the dithering beam where the materials are E-glass/Balsa wood and Carbon fiber/Balsa wood. The maximum temperature rise induced by a dithering beam on the incident face is 5753K for E-glass/balsa wood composite and 1182K for Carbon fiber/balsa wood composite. Again, Carbon fibers are better than E-glass as skin materials.

Figure 7 plots the temperature rise induced by a rotating laser beam with  $I_0 = 500W$ ,  $d = 0.0025m$ ,  $a = b = 0.05m$ ,  $L_x = L_y = 0.3048m$  in expression (3). Now, the maximum temperature rise on the incident face drops further to 963K for E-glass/balsa wood and 530K for Carbon fiber/balsa wood.

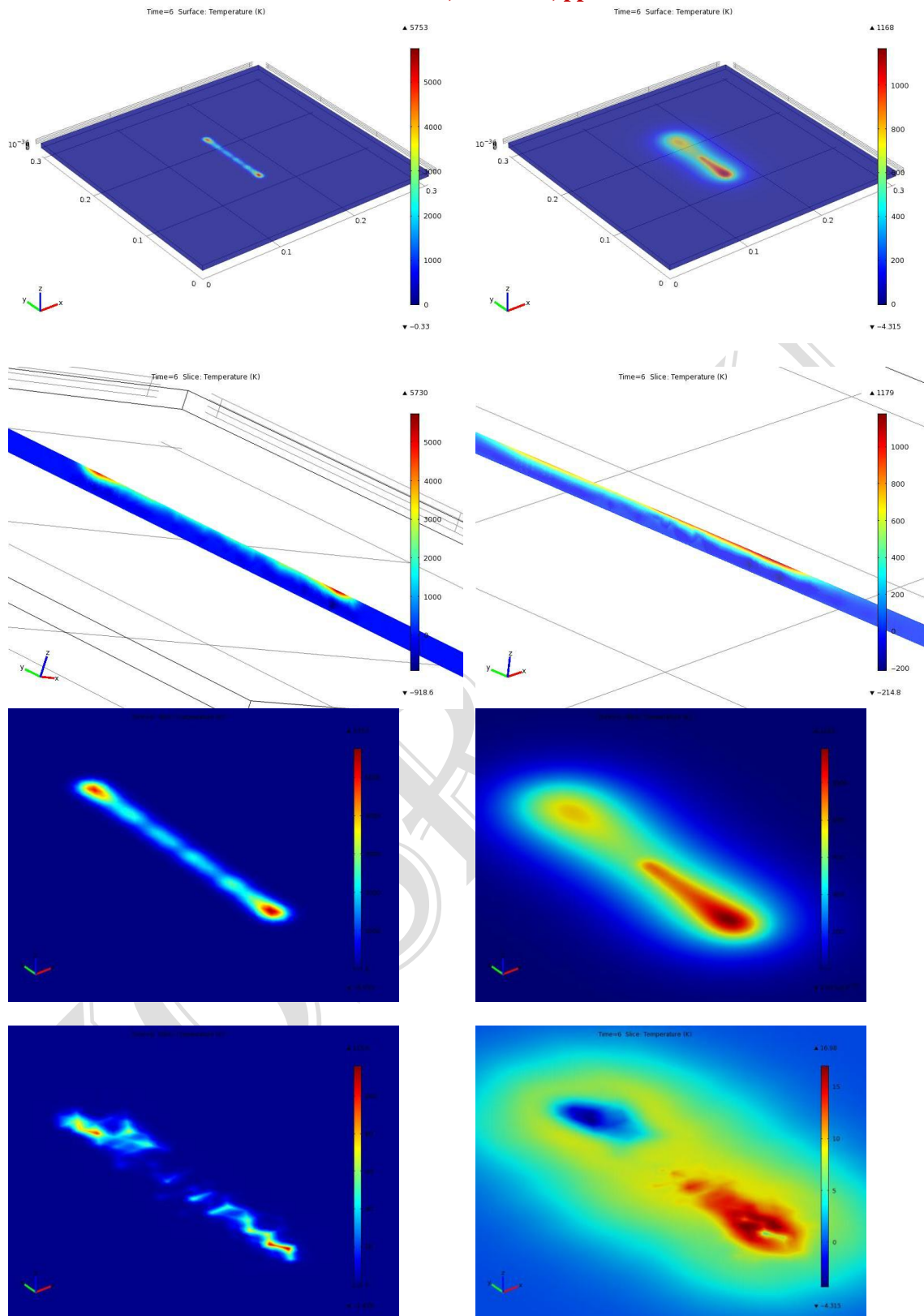
Finally, we summarize the maximum temperature rise at the top/bottom surfaces and the middle cross-section of the sandwich structure for different materials in Table 3 for a dithering beam and Table 4 for a rotating beam, respectively. It is clear from these tables that carbon fiber/honeycomb sandwich-structured composite serves best as heat shield.

#### 4. CONCLUSIONS AND FUTURE WORK

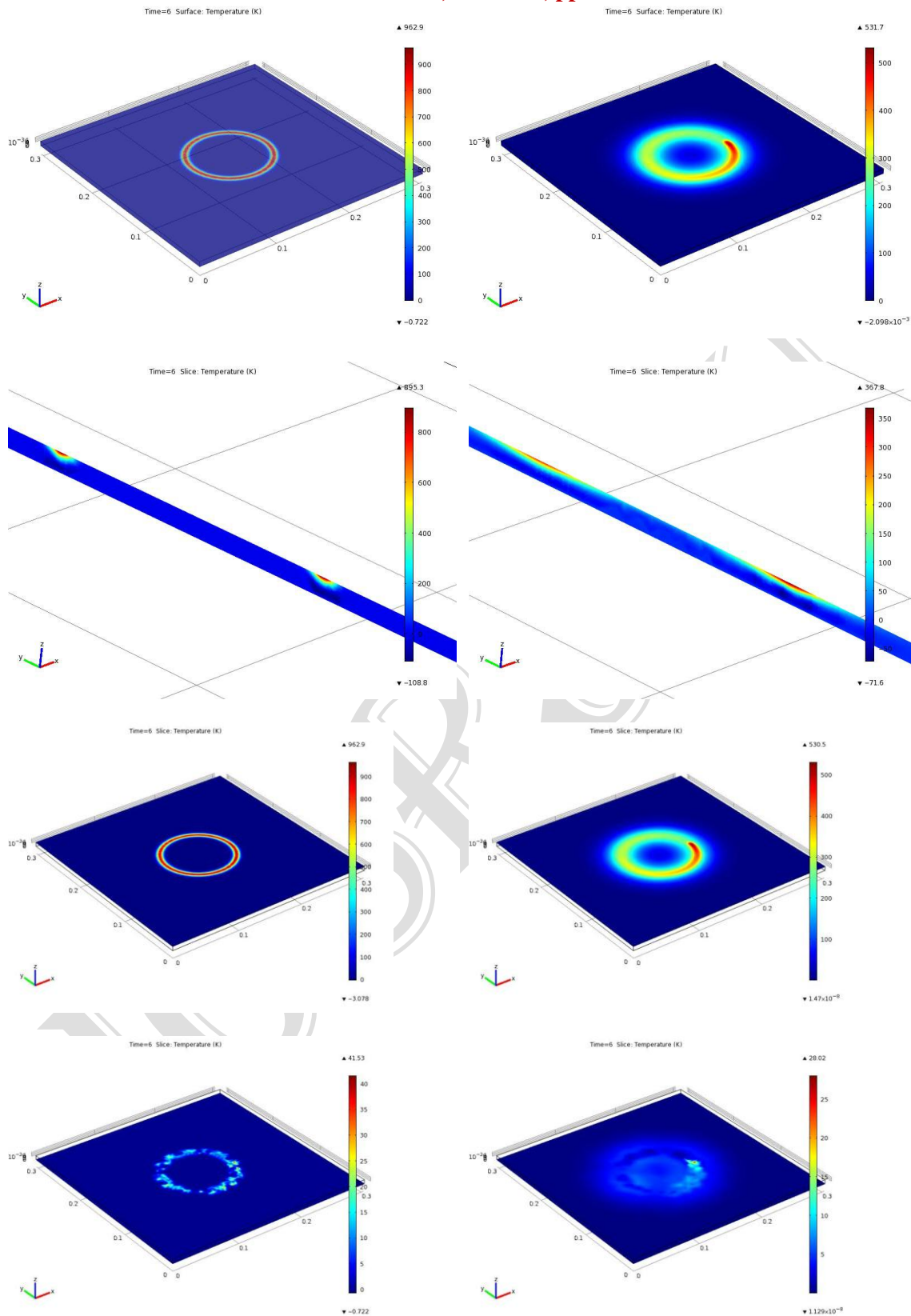
We have calculated the temperature rise induced by a rotating or dithering Gaussian laser beam for a composite sandwich structure. Our simulations indicate that carbon fibers are better skin materials than E-glass fibers and composite sandwich structure with carbon fiber skin and honeycomb core can serve extremely well as heat shields. More detailed studies on the optimal design of a composite sandwich structure are left for future exploration.

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**Figure 6: The temperature rise of a sandwich structure with Balsa wood core induced by a dithering Gaussian beam. Left column: E-glass skin; Right column: Carbon fiber skin. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.**



**Figure 7: The temperature rise of a sandwich structure with Balsa wood core induced by a rotating Gaussian beam. Left column: E-glass skin; Right column: Carbon fiber skin. Top row: 3-D view; Second row: a vertical slice through the center of the top surface; Third row: top surface; Fourth row: bottom surface.**

Table 3: Maximum temperature rise on the sandwich structure induced by a dithering Gaussian laser beam

Skin material	Core material	$T_{\max}$ on the incident face (K)	$T_{\max}$ on the middle cross-section (K)	$T_{\max}$ on the rear face (K)
E-glass fiber	Balsa wood	5753	876	115
E-glass fiber	Nomex honeycomb	3686	505	282
E-glass fiber	Cork	6090	1556	26
E-glass fiber	C-foam	4869	589	356
Carbon fiber	Balsa wood	1182	197	16
Carbon fiber	Nomex honeycomb	862	126	45
Carbon fiber	Cork	1268	369	6
Carbon fiber	C-foam	1110	143	44

Table 4: Maximum temperature rise on the sandwich structure induced by a rotating Gaussian laser beam

Skin material	Core material	$T_{\max}$ on the incident face (K)	$T_{\max}$ on the middle cross-section (K)	$T_{\max}$ on the rear face (K)
E-glass fiber	Balsa wood	963	195	41
E-glass fiber	Nomex honeycomb	653	118	87
E-glass fiber	Cork	1116	335	9
E-glass fiber	C-foam	907	143	96
Carbon fiber	Balsa wood	530	101	28
Carbon fiber	Nomex honeycomb	313	58	48
Carbon fiber	Cork	598	154	8
Carbon fiber	C-foam	432	79	50

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